Far-off-Resonant ac-Stark potential for Alkali Optical Traps

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The following text is a brief summary of our results on far-off-resonant Stark shift in alkali. We show that in the case when the light field detuning exceeds the hyper-fine splitting between the excited states the ac-Stark trapping potential reduces to a superposition of a scalar (internal state independent) shift and an effective magnetic field. We give analytical expressions for the strength of these fields

I. SYSTEM UNDER CONSIDERATION

(a) An "alkali" atom in the light field. The results described below will apply for any atom such that in the ground state

$$L_g = 0,\ S_g = 1/2,\ J_g = 1/2\,. \tag{1}$$

Here and below L is the electronic angular momentum, S is the electronic spin, J is the total electronic+spin momentum, i is the nuclear spin, and F is the total momentum;

(b) Light field has a form

$$\vec{E}(t) = \vec{E}\exp(-i\omega t) + c.c. \tag{2}$$

$$\vec{E} = E_0\,\vec{\epsilon}\,. \tag{3}$$

The factorization of the amplitude $\vec{E}$ onto a product of a scalar constant and a vector part is somewhat arbitrary. Note that the vector $\vec{\epsilon}$ does not need to have a unit length. Such a factorization will make sense in the case when the light field is spatially dependent:

$$\vec{E}(\vec{r}) = E_0\,\vec{\epsilon}(\vec{r})\,, \tag{4}$$

where the constant E_0 will play a role of some "reference" light field amplitude at some particular point in space: at this point the vector $\vec{\epsilon}(\vec{r})$ must have a unit length. (In the case of a wave-guide-based trap it will be convenient to choose E_0 to be a field amplitude in the center of the light beam "feeding" the wave-guide.);

(c) The light field detuning δ is supposed to be larger than the hyperfine splitting for the upper state manifold, but smaller then the fine splitting:

$$\Delta^e_{\mathrm{f.}} \gg \delta \gg \Delta^e_{\mathrm{h.f.}} \tag{5}$$

Below we consider both D1 and D2 line couplings.

(d) The light shifts are smaller than the hyperfine splitting between the ground states:

$$\Delta E^g \sim \frac{\hbar\Omega^2_{\mathrm{Rabi}}}{\delta} \ll \Delta^g_{\mathrm{h.f.}} \tag{6}$$

II. RESULTS

The light shift Hamiltonian for a given ground state of a momentum F_g is represented by a sum of the scalar and vector parts:

$$\hat{\mathcal{H}} = U + \mu_{\mathrm{Bohr}}\, g(L_g, S_g, J_g, i, F_g)(\hat{\vec{F}}_g \cdot \vec{H})\,. \tag{7}$$

The set of the eigenstates of the above Hamiltonian coincides with the eigenstates $|F_g, m_{F_g}\rangle$ of the projection of the total angular momentum on the direction of the effective magnetic field $\vec{H}$:

$$(\hat{\vec{F}}_g \cdot \vec{e}_{\vec{H}})|F_g, m_{F_g}\rangle = m_{F_g}|F_g, m_{F_g}\rangle. \tag{8}$$

The eigenenergies then read

$$\Delta E_{m_{F_g}} = U + \mu_{\mathrm{Bohr}}\, g(L_g, S_g, J_g, i, F_g) m_{F_g} |\vec{H}|\,. \tag{9}$$

Here

$$g(L_g, S_g, J_g, i, F_g) = g_{\mathrm{el.}}(L_g, S_g, J_g)\,\frac{F_g(F_g+1) + J_g(J_g+1) - i(i+1)}{2F_g(F_g+1)} \tag{10}$$

is the compound electronic-nuclear g-factor for the ground state under consideration,

$$g_{\mathrm{el.}}(L_g, S_g, J_g) = 1 + \frac{J_g(J_g+1) - L_g(L_g+1) + S_g(S_g+1)}{2J_g(J_g+1)} = 2 \tag{11}$$

is the Lande g-factor for the underlying electronic state ($L_g = 0$, $S_g = 1/2$, $J_g = 1/2$). The scalar field U and the effective magnetic field $\vec{H}$ have a form

$$\left.\begin{array}{rcl} U & = & +\frac{1}{3}\,U_0\,|\vec{\epsilon}|^2 \\ \mu_{\mathrm{Bohr}}\vec{H} & = & -\frac{2}{3}\,U_0\,[\Re(\vec{\epsilon}^{\,*}) \times \Im(\vec{\epsilon})] \end{array}\right\}\ \mathrm{D1}\,(J_e = 1/2) \tag{12}$$

$$\left.\begin{array}{rcl} U & = & +\frac{2}{3}\,U_0\,|\vec{\epsilon}|^2 \\ \mu_{\mathrm{Bohr}}\vec{H} & = & +\frac{2}{3}\,U_0\,[\Re(\vec{\epsilon}^{\,*}) \times \Im(\vec{\epsilon})] \end{array}\right\}\ \mathrm{D2}\,(J_e = 3/2)\,, \tag{13}$$

where

$$U_0 = \frac{\langle J_e||d||J_g\rangle^2 |E_0|^2}{(2J_e+1)\hbar\delta} \tag{14}$$

is a "reference" constant controlling the overall amplitude of the field.

The constants U_0 are, in fact, the same for both D1 and D2 lines: they can be expressed through the corresponding reduced dipole matrix element of the "under-underlying" transition $L_g = 0 \leftrightarrow L_e = 1$

$$U_0 = \frac{\langle L_e||d||L_g\rangle^2 |E_0|^2}{(2L_e+1)\hbar\delta} \tag{15}$$

$$= \frac{\langle L_e||d||L_g\rangle^2 |E_0|^2}{3\hbar\delta} \,. \tag{16}$$

On the other hand we note that the constant U_0 is nothing else but the energy shift of the $F_g = F_{g,\,max} = J_g + i$, $m_{F_g} = F_{g,\,max}$ level coupled by a σ_+ light field of an amplitude E_0 with the $F_e = F_{e,max} = F_{g,\,max} + 1$, $m_{F_e} = +F_{e,max}$ excited state ("closed transition"):

$$U_0 = \frac{\langle F_e||d||F_g\rangle^2 |E_0|^2}{(2F_e+1)\hbar\delta} \,. \tag{17}$$

This observation makes it easy to express the constant U_0 through the corresponding resonant saturation parameter and the Doppler energy:

$$U_0 = \frac{1}{8}\left(\frac{I_0}{I_{\rm sat}}\right)\left(\frac{\Gamma}{\delta}\right)\hbar\Gamma \,, \tag{18}$$

where I_0 is the field intensity corresponding to the amplitude "reference" E_0, $I_{\rm sat}$ is the saturation intensity for the above closed transition. We have used the following definition for the saturation parameter $s = I_0/I_{\rm sat}$:

$$s = \frac{\Omega_{\rm Rabi}^2/2}{\Gamma^2/4} \,, \tag{19}$$

where

$$\Omega_{\rm Rabi} = -2\,\frac{\langle F_e||d||F_g\rangle E_0}{\sqrt{2F_e+1}} \tag{20}$$

is the corresponding Rabi frequency.